

# CAUSAL INFERENCE FOR SURVIVAL OUTCOMES: A CENSORED EDITION

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# Formulating Causal Questions

(Goetghebeur et al, 2020, Stat in Med)

**To get a causal answer we need to start with a causal question!**

1. Define the treatment
2. Define the outcome
3. Specify population(s) of interest
4. Formalise potential outcomes (POs)
5. Specify target causal effect, i.e. the *estimand*, as a (summary) contrast between the PO-distributions
6. Assumptions identifying estimand from available data
7. Statistical inference with suitable methods
8. Evaluate plausibility of assumptions / sensitivity analyses

# Target Trial

(Parra et al, 2020, arXiv:2011.11771)



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## A general principle to elicit & specify a causal question

- The ideal (hypothetical) trial that would answer the research question
  - possibly disregarding practical, ethical, financial constraints
  - ... but not disregarding laws of physics (no “turning back time”)
  
- Especially useful in time-dependent situations
  - fix time ‘zero’
  - prevent immortal time bias etc.

# Motivating Example



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- End-stage renal disease: which renal replacement therapy (RRT)?
- Pre-emptive transplant (PKT) vs “start with dialysis”?
  - binary point treatment
  - a bit like ITT
- Wanted: “effect” on time to all-cause mortality starting from RRT
  - exact definition of estimand?
  - no competing events here
- *Most studies on the topic suffer from avoidable biases*  
(Parra et al, 2020 arXiv:2011.11771)

# Motivating Example

## Aim:

- Want to mimic RCT (*target trial*):  
as if individuals randomised to treatment (PKT) / control (dialysis)
  - need plausible assumptions
  - & suitable methods

# Causal Inference - Basics

- $A$  = binary **point** treatment
- $Y$  = outcome (general)
- $Y_a$  = potential outcome if we **set**  $A = a$  by (well-defined) intervention
- Common causal contrasts (**estimands**):
  - (total) average effect:  $E(Y_1) - E(Y_0)$
  - effect on the treated:  $E(Y_1 | A=1) - E(Y_0 | A=1)$

# Causal Inference - Basics

- Assumptions:

- Causal consistency & positivity, no interference
- **No U**nmeasured **C**onfounding (**NUC**)
- Some (semi-)parametric model

⇒ Identification

- Many methods for estimation

- outcome regression, stratification / matching, IPTW, DR
- with sufficient set of covariates, possibly summarised in propensity score
- check: overlap and balance!

# Now: Survival Outcome

- Outcome  $Y = T$  = time-to-event
  - What's different?
    - Censoring
      - for some units we only know: the event did not occur in some period
    - Dynamics – things happen over time (including treatment)
      - 'mean' not a good summary?
- ⇒ May want different causal estimands
- assumptions?
  - methods?

## Desirable estimand?

- Risk differences at relevant times

$$P(T_1 > t) - P(T_0 > t), \quad t \text{ in } [0, \mathcal{T}]$$

- i.e. difference in (*marginal*) survival functions of POs
- Interpretation: risk difference for no event by time  $t$  had random patient been treated versus not
  - ≈ total average causal effects for meaningful time points
  - could easily be by baseline subgroups (*no further details today*)

# Survival Outcome - Estimands

**Hazard scale?** Hazard ratio (HR) / contrast of hazards - popular

- With potential outcomes:

$$\lambda_a(t) = \lim_{h \rightarrow 0} \frac{1}{h} P( t \leq T_a < t+h \mid T_a \geq t )$$

- i.e. hazard function in arm 'a' of our target trial
- Contrasts of  $\lambda_1(t)$  vs.  $\lambda_0(t)$  are conditional on possibly different 'subgroups'  $\{T_1 \geq t\}$  and  $\{T_0 \geq t\}$ 
  - survivors **at a given time  $t$**  in the two arms not necessarily comparable anymore even in an RCT

# Risks & Hazards – Pros and Cons

- Difficult to interpret causal effects on the hazard scale **correctly**
  - no such thing as ‘the’ causal effect
  - ‘effect reversals’ between hazard and risk scale *possible*  
*(Martinussen et al., 2020 LIDA)*

⇒ must be aware & take into account for correct interpretation of contrasts of hazards
  
- But  $\lambda_a(t)$  as whole function of  $t$ : one-to-one relation with  $P(T_a > t)^*$ 

⇒ hazards still useful modelling tool (*+ model checking etc. well-established*)

  - especially to deal with censoring & include relevant covariates

*\*in absence of comp.events*

# Estimands - Summary

- We like & recommend contrasts on risk scale
  - direct clinical interpretation
  - but may use hazard models as a tool to get there
- There may sometimes be specific reasons to choose hazard contrasts as causal estimands...
  - ... but don't let it be just by 'default' or because 'everyone does it'
- Many other estimands – *not enough time today*
  - 'speed' scale (accelerated failure time models) – useful for time-varying treatments
  - restricted mean survival time etc.

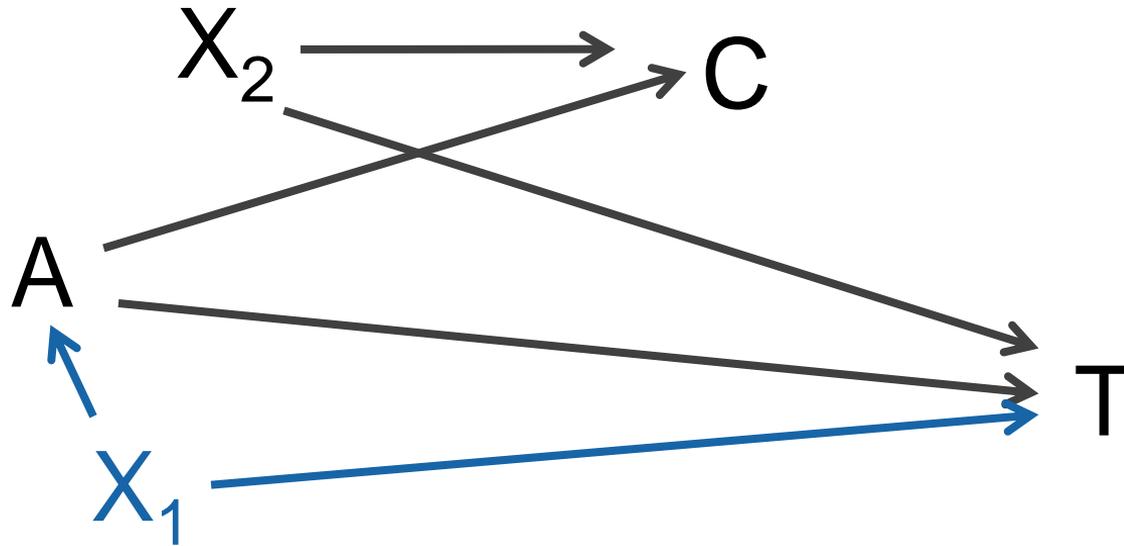
## What about censoring?

- Want estimand ‘outside’ of a study setting, i.e. ‘without’ censoring
- **Target trial:** has no censoring at all  
or at least same complete follow-up  $\mathcal{T}$  for everyone
  - aka ‘elimination of censoring’, or ‘*complete populations*’ (TG8)
  - careful with special ‘censoring’ events: drop-out, treatment switching, competing events
    - *relevant target trial **without** these types of intercurrent events?*
  - similar **reasoning & assumptions** as with counterfactual treatment!
    - *in particular: think about common causes of censoring and outcome event*

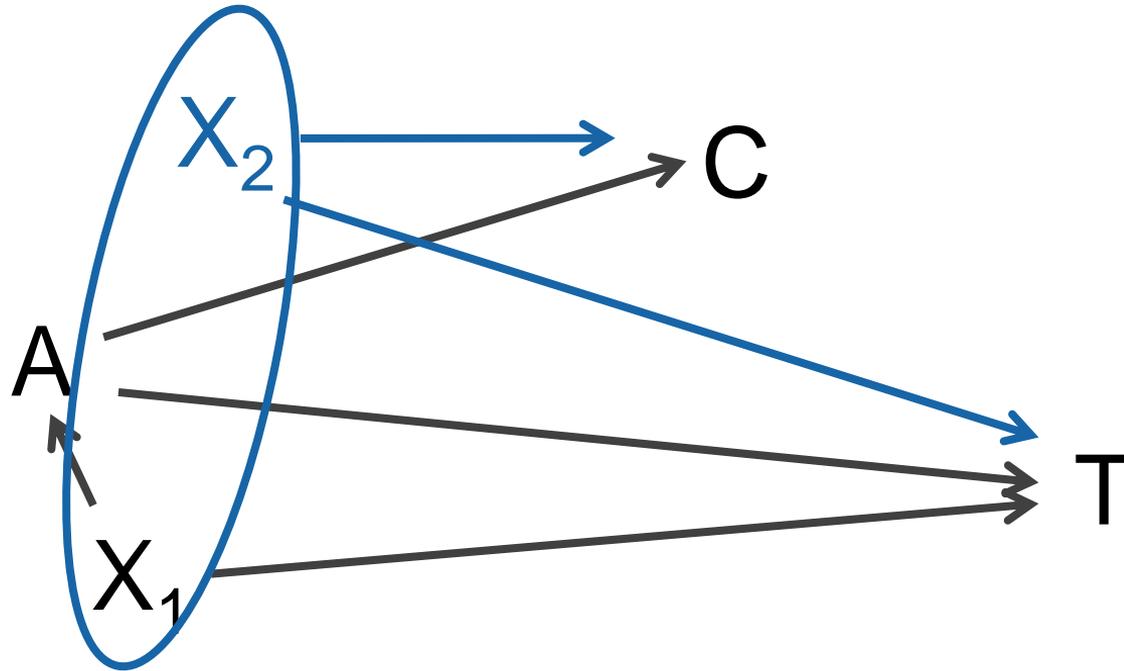
# Key Assumptions

- **NUC:**  $X_1$  = sufficient covariate information regarding treatment assignment confounding
- $X_2$  = sufficient covariate information regarding possibly (time-varying) 'common causes' of censoring and event  
≈ ensure 'conditionally independent' censoring (*TG8*)
- Methods must use  $X_1$  &  $X_2$  jointly
  - ( *$X_1$  and  $X_2$  can overlap*)

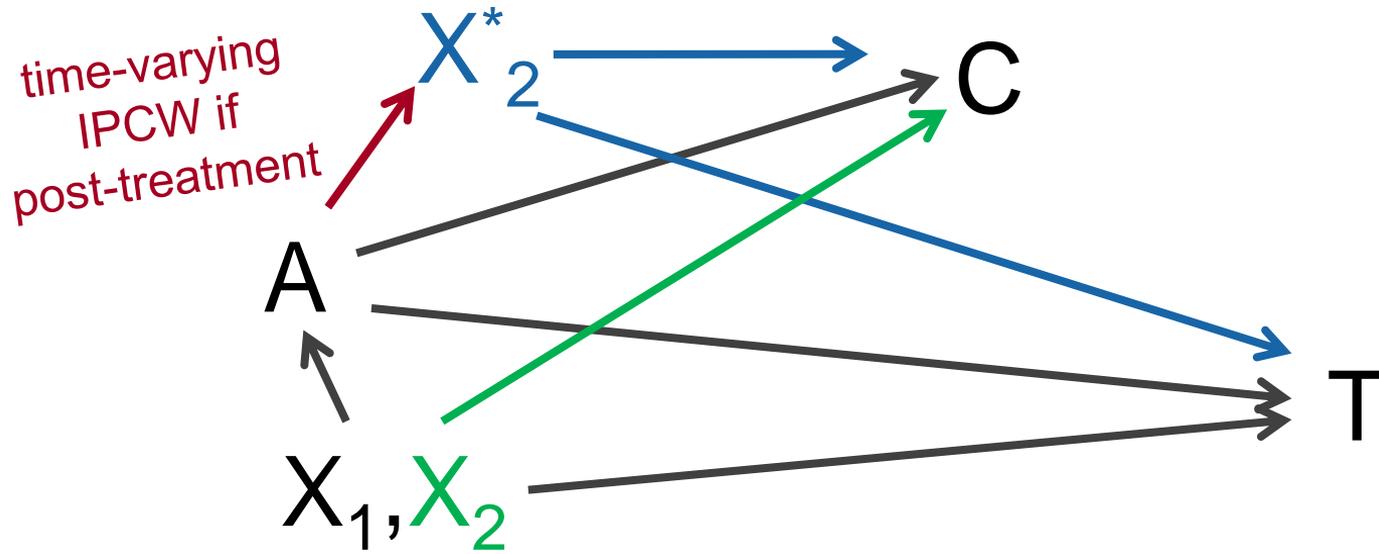
# Key Assumptions - DAG



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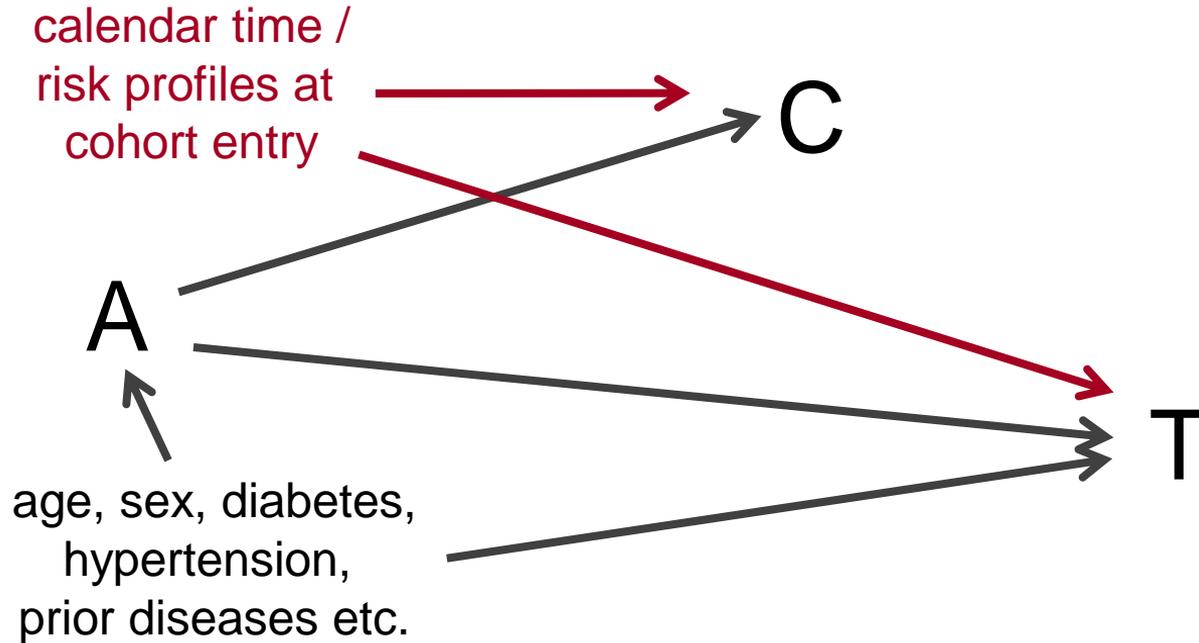


# Key Assumptions - DAG



# RRT – PKT Example

(Parra et al, 2020, arXiv:2011.11771)



## Model-based **marginal** counterfactual survival curves:

- (Sufficiently) flexible hazard models
  - possibly separately for treatment groups
  - include both sets of baseline covariates  $X_1$  and  $X_2$

+ transformation to risk scale

**+ standardisation**

*Software:*

- R: stdReg (*Sjolander & Dahlqwist*); Stata: stpm2\_standsurv (*Lambert*)
- discrete-time-methods: plenty of code (*Hernan & Robins, book*)

## Weighted Kaplan-Meier curves:

- Inverse probability of **treatment & censoring** weighting
- Note: including covariates in **IPTW** does not suffice if also needed to adjust for confounding of censoring

⇒ need **IPCW** too (time-varying)      (*Roysland, Didelez et al., 202?- tba*)

### Software:

- R: survival library (*Therneau*), R: ipw (*van der Wal et al, 2011*), R: ahw (*Ryalen, github*); core Stata

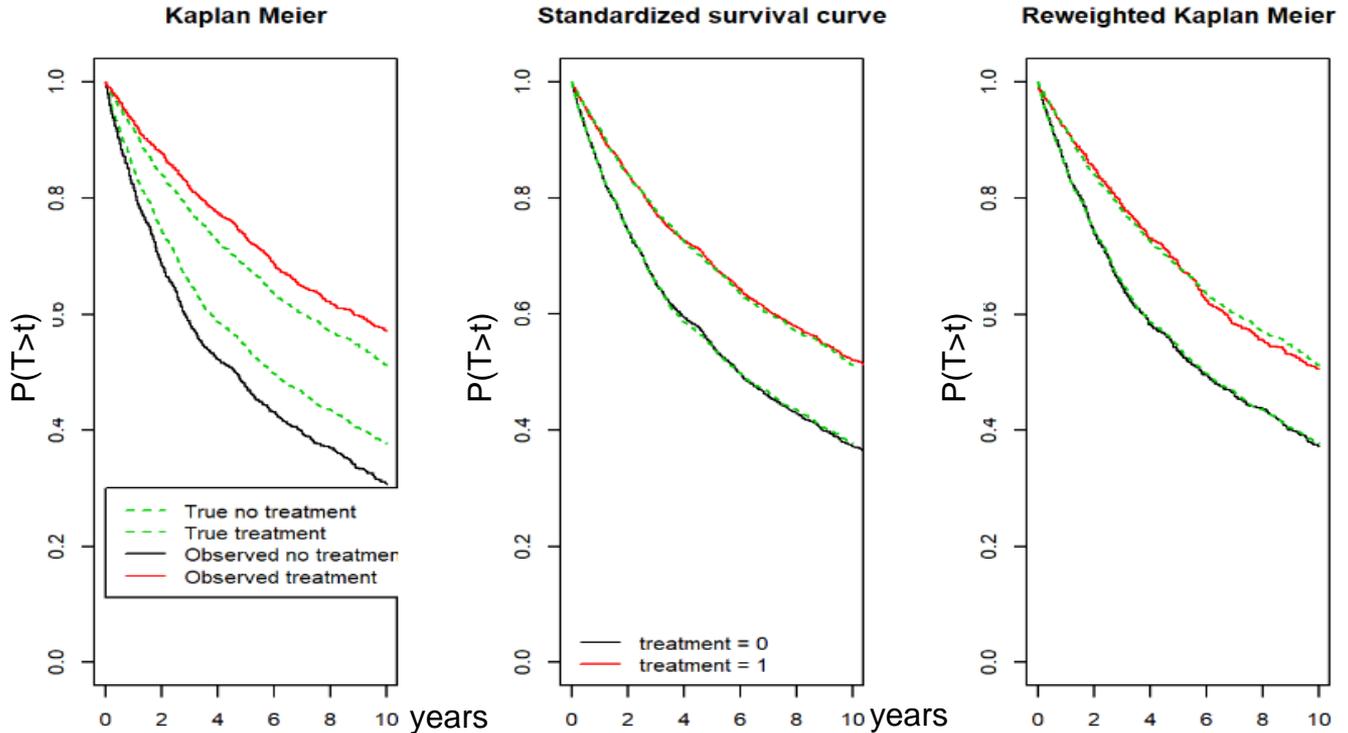
# Illustration

Simulated data inspired by RRT data (but somewhat simplified)

N=2000

Confounding  
by observed  
covariates  
& no censoring

(very basic  
programming)



# Illustration

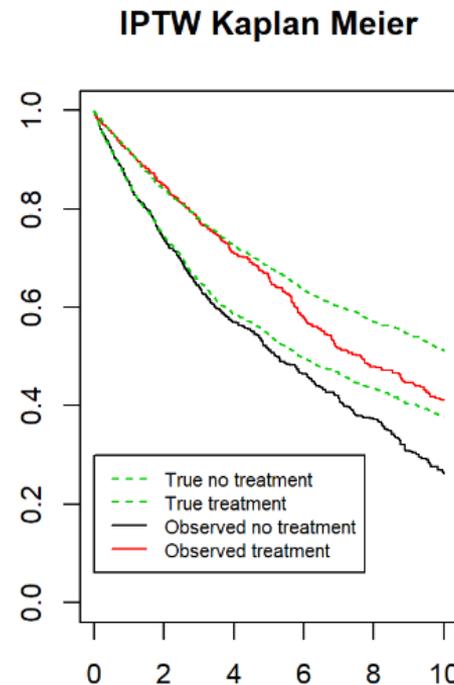
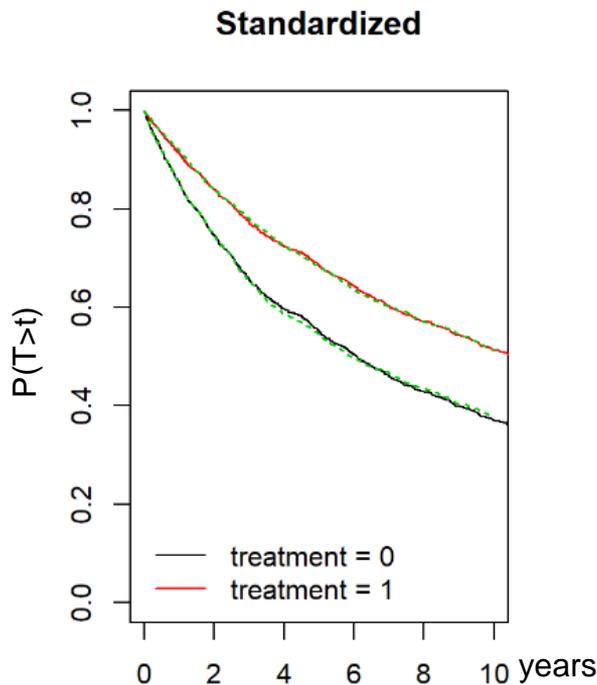
Simulated data inspired by RRT data (but somewhat simplified)

N=2000

Confounding  
by observed  
covariates  
& with censoring

Only using IPTW  
not good enough

Some improvement  
with IPCW (not shown)



## Key messages

- Can & should choose meaningful, clinically relevant causal estimands for survival outcomes
  - **target trial** should also address **censoring**
- Hazard models well-established – only need to be suitably transformed
- Think ‘causally’ about censoring to justify key assumptions
  - in addition to ‘no unmeasured confounding’
- More details on (simple) implementation / software in paper – forthcoming!

# STRATOS

INITIATIVE



## Contact

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